Christ Church Grammar School

2019
TEST 2

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name_____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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Instructions to candidates

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3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

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6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(9 marks)**

Evaluate each of the following integrals (Leave answers with positive indices):

(a) $\int \frac{2}{\sqrt{x}} - \sqrt[3]{x}\ dx$ (2 marks)

(b) $\int \frac{x^3-1}{(x^4-4x)^3} dx$ (3 marks)

See next page

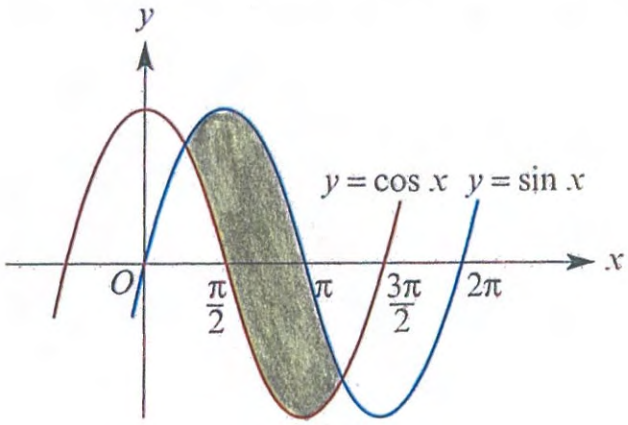
Question 1 continued

(c) Find the exact area between the curve $y = x^2 - 3$ and the x –axis. Include a diagram as part of your solution. (4 marks)

See next page

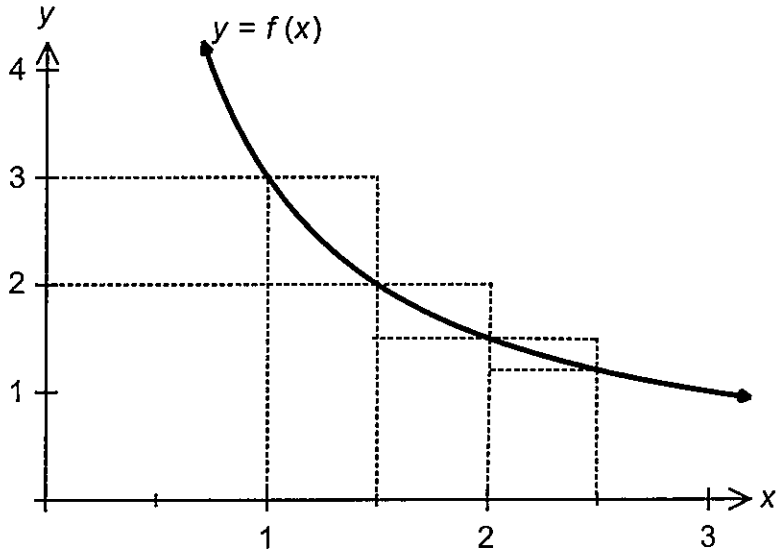
Question 2 **(3 marks)**

Find the exact area of the shaded region below.



Question 3 **(3 marks)**

Consider the graph of $f(x)$, and the table of values of $f(x)$ for certain x values.



x	1	1.5	2	2.5
$f(x)$	3	2	1.5	1.2

By considering the rectangles drawn on the graph, calculate the value of p and q, such that $p < \int_1^{2.5} f(x)\,dx < q$, where p and q represent the under and over estimates respectively.

End of questions

Additional working space

Question number: _____

Christ Church Grammar School

2019
TEST 2

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

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See next page

Question 4 **(10 marks)**

The acceleration of an object undergoing rectilinear motion is given by $a = 3t + 5\ ms^{-2}$. The object has an initial velocity of 20 m/s and begins its motion at a displacement of $-10\ m$.

(a) Determine:

(i) the velocity at t = 3 seconds. (3 marks)

(ii) the displacement at t = 3 seconds. (3 marks)

(b) Show that the object is never at rest. (2 marks)

(c) Hence, or otherwise, determine the distance travelled by the object in the first three seconds. (2 marks)

See next page

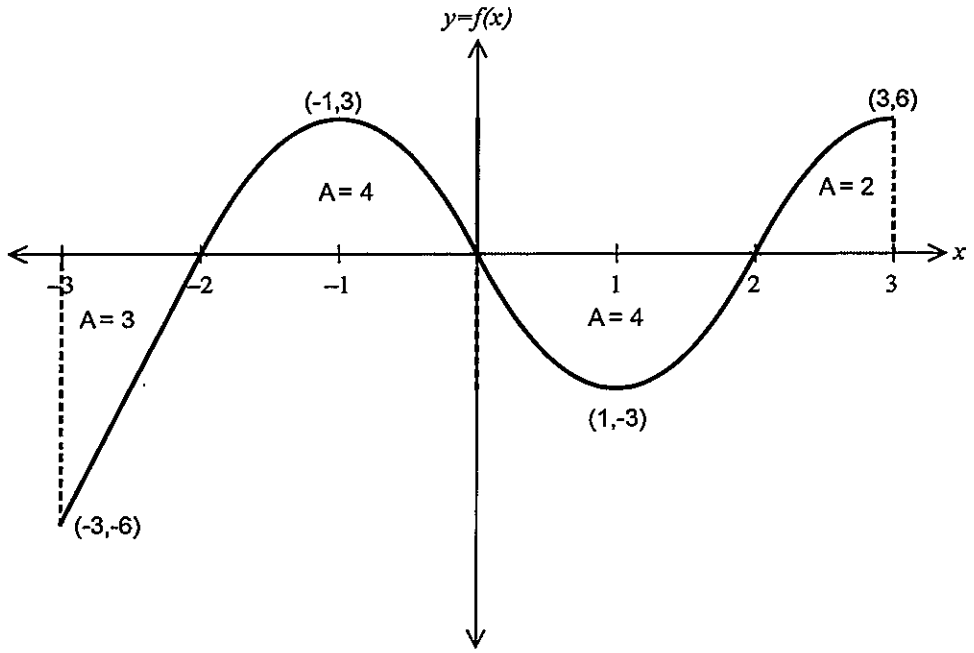
Question 5 **(3 marks)**

Calculate the area of the regions enclosed by the curves $y = \cos(x)$and $y = \sin(2x)$ for $0 \leq x \leq \frac{\pi}{2}$. Include a sketch as part of your solution.

See next page

Question 6 **(7 marks)**

The graph of the function $f(x)$ is shown below for $-3 \leq x \leq 3$. The areas, (A), enclosed between the graph, the x-axis and the lines $x = -3$ and $x = 3$ are marked in the appropriate regions.



Determine:

(a) (i) the value of $\int_{-3}^{2} f(x)\,dx$ (2 marks)

(ii) the area enclosed between the graph of $f(x)$ and the x axis, from $x = -3$ to $x = 2$. (2 marks)

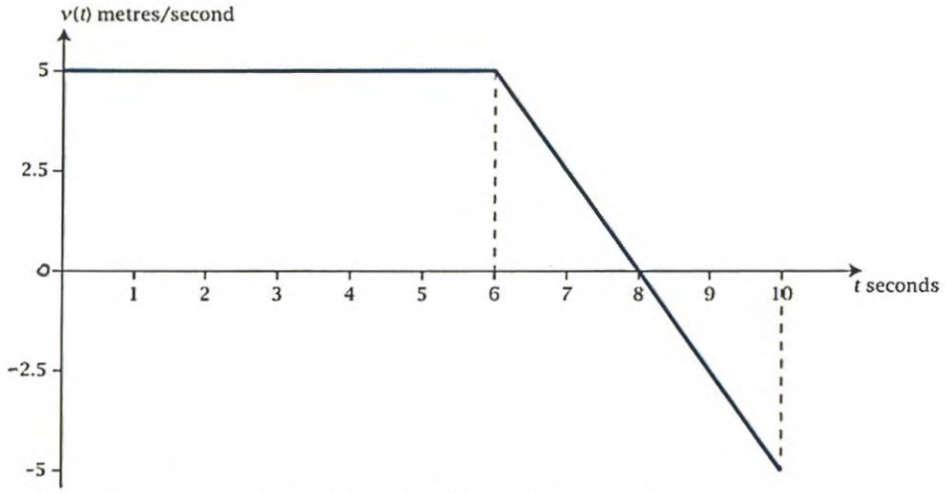
Question 6 continued

(b) $\int_2^0 (x - f(x))\, dx$ (3 marks)

See next page

Question 7 **(7 marks)**

A particle moves along a straight line. The velocity – time graph is shown below.



(a) Find the velocity of the particle when $t = 3$. (1 mark)

(b) Write an expression for v in terms of t for $6 \leq t \leq 10$. (2 marks)

(c) Find the acceleration of the particle when

(i) $t = 1$ (1 mark)

(i) $t = 7$ (1 mark)

(d) Find the change in displacement for $0 \leq t \leq 10$. (2 marks)

See next page

Question 8 **(3 marks)**

The area enclosed by the line $y = mx$ and the parabola $y = x^2$ is 24.813. Find the value of m, correct to 1 decimal place, where $m > 0$.

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Christ Church Grammar School

2019
UNIT TEST 2

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Student name — SOLUTIONS —

Teacher name _____

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Question 1 **(9 marks)**

Evaluate each of the following integrals (Leave answers with positive indices):

(a) $\int \frac{2}{\sqrt{x}} - \sqrt[3]{x}\, dx$ (2 marks)

$\int 2x^{-1/2} - x^{1/3}\, dx$

$= \frac{2x^{1/2}}{1/2} - \frac{x^{4/3}}{4/3} + c$ ✓ (Integrates correctly)

$= 4x^{1/2} - \frac{3x^{4/3}}{4} + c$ ✓ (positive indices)

(b) $\int \frac{x^3-1}{(x^4-4x)^3} dx$ (3 marks)

$= \frac{1}{4} \int \frac{4x^3-4}{(x^4-4x)^3} dx$ ✓ (Adjusts numerator)

$= \frac{1}{4} \times \frac{1}{-2} (x^4-4x)^{-2} + c$ ✓

$= -\frac{1}{8(x^4-4x)^2} + c$ ✓ (positive indices)

If $y = (x^4-4x)^{-2}$ $\left(\times \frac{1}{-2}\right)$.

$\frac{dy}{dx} = -2(x^4-4x)^{-3}(4x^3-4)$

$= \left(\times \frac{1}{-2}\right) \frac{-2(4x^3-4)}{(x^4-4x)^3}$

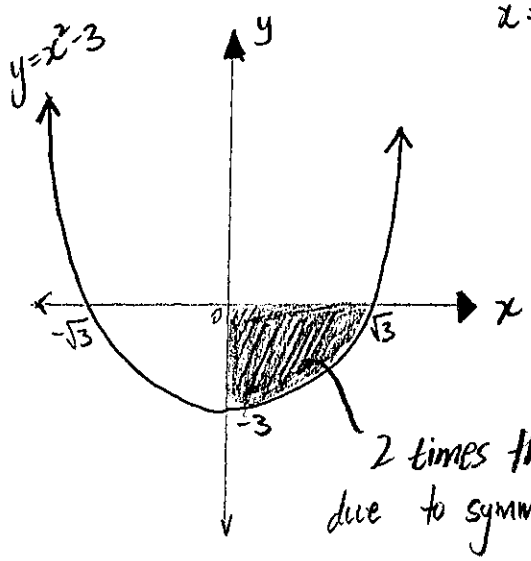
5

Question 1 continued

(c) Find the exact area between the curve $y = x^2 - 3$ and the x –axis. Include a diagram as part of your solution. (4 marks)

$0 = x^2 - 3$

$x = \pm\sqrt{3}$



(Diagram) ✓

2 times this area due to symmetry of $y=x^2-3$

(Correct Integral) ✓

$\therefore 2 \times \int_0^{\sqrt{3}} x^2 - 3 \, dx$

(Integrates Correct) ✓

$= \left[\frac{x^3}{3} - 3x\right]_0^{\sqrt{3}}$

$= 2 \times \left(\frac{3\sqrt{3}}{3} - \frac{3\sqrt{3}}{1} - 0\right)$

$= 2 \times \left(-\frac{6\sqrt{3}}{3}\right)$

$\therefore$ Area $= 4\sqrt{3}$ units^2 ✓

OR

$\int_{-\sqrt{3}}^{\sqrt{3}} x^2 - 3 \, dx$

$= \left[\frac{x^3}{3} - \frac{3x}{1}\right]_{-\sqrt{3}}^{\sqrt{3}}$

$= \left[\frac{x^3 - 9x}{3}\right]_{-\sqrt{3}}^{\sqrt{3}}$

$= \frac{3\sqrt{3} - 9\sqrt{3}}{3} - \left(\frac{-3\sqrt{3} + 9\sqrt{3}}{3}\right)$

$= \frac{6\sqrt{3} - 18\sqrt{3}}{3}$

$= \frac{-12\sqrt{3}}{3}$

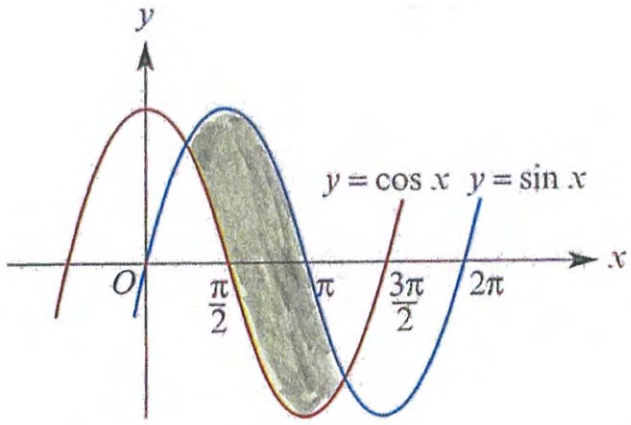
$\therefore$ Area $= 4\sqrt{3}$ units^2

4

See next page

Question 2 **(3 marks)**

Find the exact area of the shaded region below.



$\cos x = \sin x$ when $x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\therefore \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin x - \cos x \, dx$ ✓ (Sets up integral)

$= \left[-\cos(x) - \sin x \right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$ ✓ (Antidifferentiates)

$= -\cos\left(\frac{5\pi}{4}\right) - \sin\left(\frac{5\pi}{4}\right) - \left[-\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right) \right]$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right)$

MUST KNOW EXACT VALUES!

$= \frac{4\sqrt{2}}{2}$

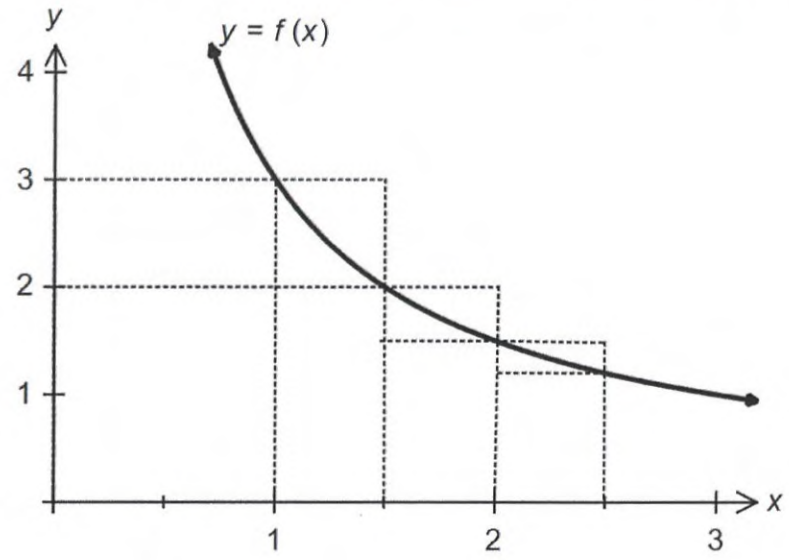
Area $= \underline{2\sqrt{2}}$ square units. ✓ (Correct Area)

3

See next page

Question 3 **(3 marks)**

Consider the graph of $f(x)$, and the table of values of $f(x)$ for certain x values.



x	1	1.5	2	2.5
$f(x)$	3	2	1.5	1.2

By considering the rectangles drawn on the graph, calculate the value of p and q, where:

$$p < \int_1^{2.5} f(x)\,dx < q$$

$p = 0.5[2 + 1.5 + 1.2]$; $q = 0.5[3 + 2 + 1.5]$

$p = \frac{1}{2}(4.7)$ $q = \frac{1}{2}(6.5)$

$p = 2.35$ ✓ $q = 3.25$ √√

[1mk for one correct
3mks for both with
working]

End of questions

3

Additional working space

Question number: _____

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UNIT TEST 2

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Student name _____

Teacher name _____

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Question 4 **(10 marks)**

The acceleration of an object undergoing rectilinear motion is given by $a = 3t + 5\ ms^{-2}$. The object has an initial velocity of 20 m/s and begins its motion at a displacement of $-10\ m$.

(a) Determine:

(i) the velocity at t = 3 seconds. (3 marks)

$v(t) = \frac{3t^2}{2} + 5t + c$

but $v(0) = 20$

$\therefore 20 = c$ ✓

$v(t) = \frac{3t^2}{2} + 5t + 20$ ✓

$v(3) = 48.5\ m/s.$ ✓

(ii) the displacement at t = 3 seconds. (3 marks)

$x(t) = \int \frac{3t^2}{2} + 5t + 20\ dt$ ✓ $\Rightarrow \frac{t^3}{2} + \frac{5t^2}{2} + 20t - 10$ ✓

$\therefore x(3) = 86m$ ✓

(b) Show that the object is never at rest. (2 marks)

At rest if $v = 0$ ✓

but $\frac{3t^2}{2} + 5t + 20 \neq 0, \forall t$ ✓

OR

$\Delta < 0$

$\Delta = -95.$

Hence, or otherwise,

(c) determine the distance travelled by the object in the first three seconds. (2 marks)

At $t=0$ object @ $-10m$

$t = 3$ object @ $86m$

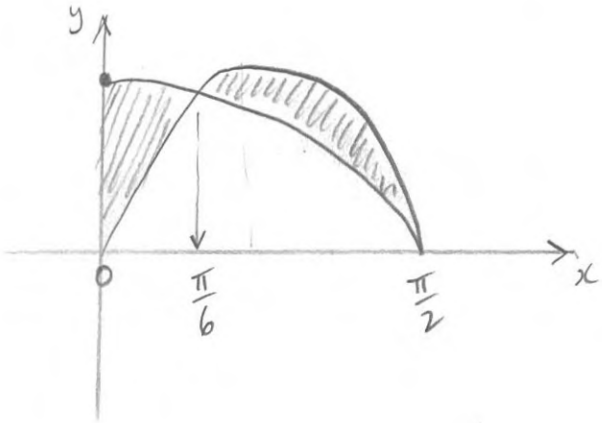
} No turns between ✓

$\therefore$ Dist in 1st 3sec = $96m$ ✓

10

Question 5 **(3 marks)**

Calculate the area of the region enclosed by the curves $y = \cos(x)$ and $y = \sin(2x)$ for $0 \leq x \leq \frac{\pi}{2}$. Include a sketch as part of your solution.



✓ (sketch)

$$\text{Area} = \int_0^{\pi/6} \cos(x) - \sin(2x)\, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin(2x) - \cos(x)\, dx$$

✓ (Integral)

$$\text{Area} = \frac{1}{4} + \frac{1}{4}$$

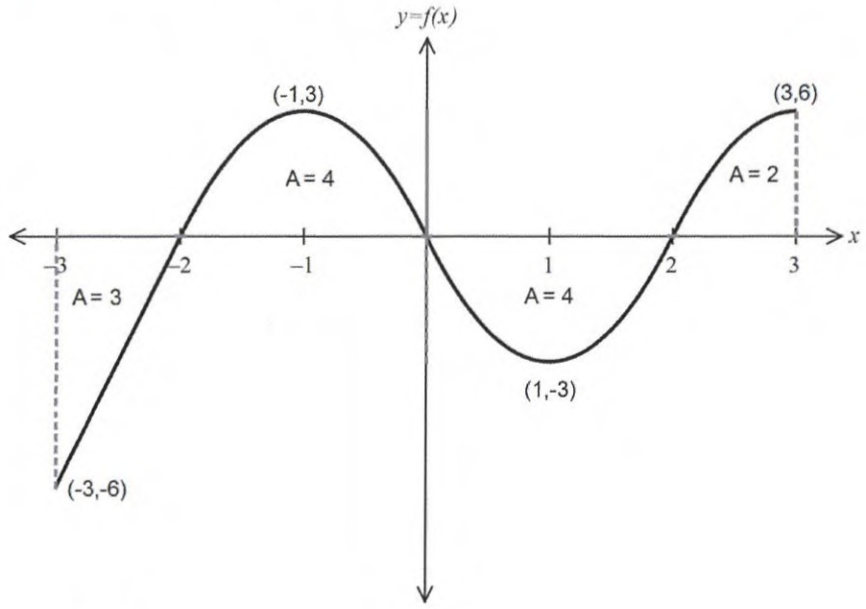
$$\text{Area} = \frac{1}{2} \text{ units}^2$$

✓ (Answ.)

3

Question 6 **(7 marks)**

The graph of the function $f(x)$ is shown below for $-3 \leq x \leq 3$. The areas, (A), enclosed between the graph, the x-axis and the lines $x = -3$ and $x = 3$ are marked in the appropriate regions.



Determine:

(a) (i) the value of $\int_{-3}^{2} f(x)\,dx$ (2 marks)

$\int_{-3}^{2} f(x)\,dx = -3+4-4$

$= -3$ ✓✓

(ii) the area enclosed between the graph of $f(x)$ and the x axis, from $x = -3$ to $x = 2$. (2 marks)

Area $= 3+4+4$

$= 11$ units^2 ✓✓

4

Question 6 continued

(b) $\int_2^0 (x - f(x))\, dx$ (3 marks)

$= \int_2^0 x\ dx - \int_2^0 f(x)\ dx$ ✓ (Separates Integral)

$= -\int_0^2 x\ dx + \int_0^2 f(x)\ dx$

$= -\left[\frac{x^2}{2}\right]_0^2 - 4$ ✓ (Integrates x)

$= -\left(\frac{4}{2} - 0\right) - 4$

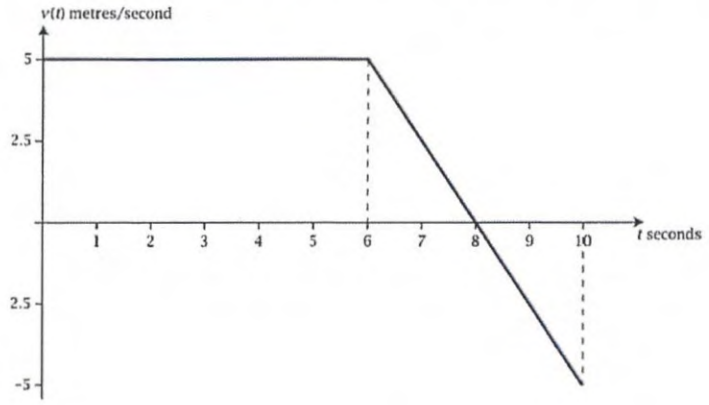
$= -2 - 4$

$= -6$ ✓ (Correct Answer)

3

Question 7 **(7 marks)**

A particle moves along a straight line. The velocity – time graph is shown below.



(a) Find the velocity of the particle when $t = 3$. (1 mark)

$t=3$, $v = 5\ m/s$ ✓

(b) Write an expression for v in terms of t for $6 \leq t \leq 10$. (2 marks)

Equation of line $v(t) = -2.5t + 20\ m/s$ ✓ ✓

(c) Find the acceleration of the particle when

(i) $t = 1$ (1 mark)

$a = 0\ ms^{-2}$ ✓

(i) $t = 7$ (1 mark)

$a = -2.5\ ms^{-2}$ ✓

(d) Find the change in displacement for $0 \leq t \leq 10$. (2 marks)

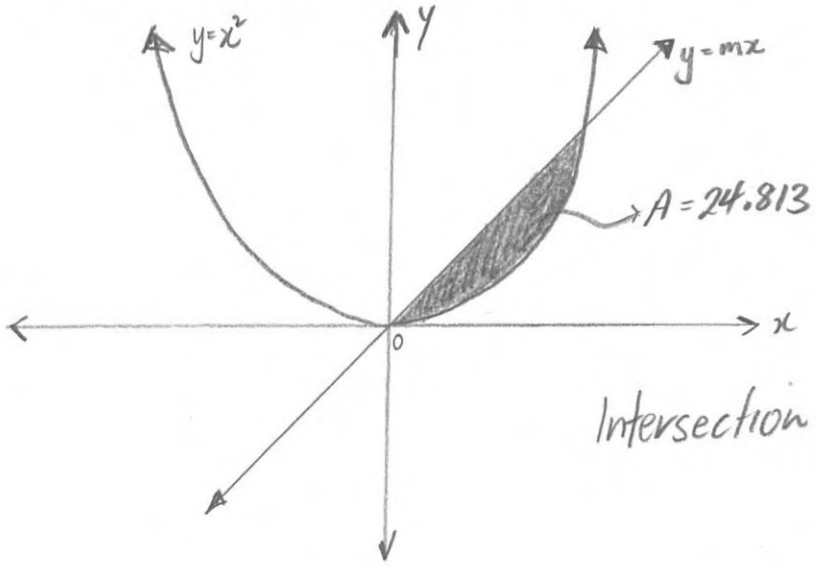
Net Area under curve = change in displacement

$\therefore 5 \times 6 = 30m$ (as triangles cancel out) ✓✓

7

Question 8 **(3 marks)**

The area enclosed by the line $y = mx$ and the parabola $y = x^2$ is 24.813. Find the value of m, correct to 1 decimal place, where $m > 0$.



Intersection $x^2 = mx$

$x^2 - mx = 0$

$x(x-m) = 0$

$\therefore x = 0, m$ ✓ (intersection)

Hence $\int_0^m (mx - x^2)\,dx = 24.813$ ✓ (Equation)

Using ClassPad $m = 5.3$ ✓ (m to 1dp)

3

End of questions